Exercise: 10.1

1. How many tangents can a circle have?

Sol. A circle can have infinitely many tangents.

2. Fill in the blanks:

(i) A tangent to a circle intersects it in .... point(s).

Sol. one

(ii) A line intersecting a circle in two points is called a ....

Sol. Secant

(iii) A circle can have .... parallel tangents at the most.

Sol. two

(iv) The common point of a tangent to a circle and the circle is called ....

Sol. point of contact

3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

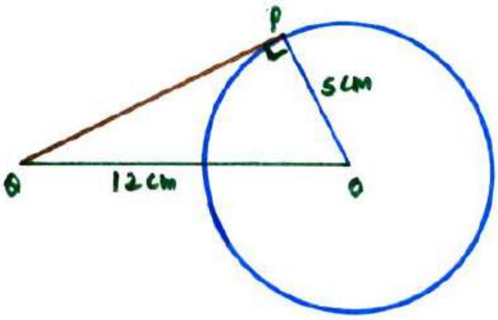
(A) 12 cm

(B) 13 cm

(C) 8.5 cm

(D) $\sqrt{119}$ cm

Sol. The line drawn from the centre of the circle to the tangent is perpendicular to the tangent.



$\therefore$ OP $\perp$ PQ

In $\perp$ ΔOPQ

$OP^2 + PQ^2 = OQ^2$ [By Pythagoras theorem]

$\Rightarrow 5^2 + PQ^2 = (12)^2$

$\Rightarrow PQ^2 = 144 - 25$

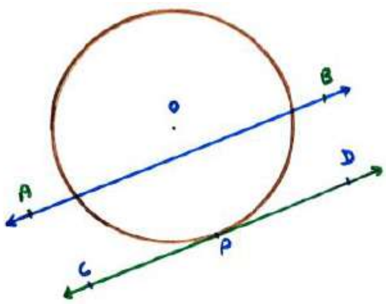
$\Rightarrow PQ^2 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

$\therefore$ (D) is the correct option.

4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Sol. AB and CD are two parallel lines where CD is the tangent to the circle at point C while AB is the secant to the circle.



Exercise: 10.2

In Q.1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm
(C) 15 cm (D) 24.5 cm

Sol. Here P is the point of contact

$\therefore OP \perp PQ$

$\therefore$ In $\perp \Delta POQ$,

$OP^2 + PQ^2 = OQ^2$

[By Pythagoras theorem]

$\Rightarrow OP^2 + 24^2 = 25^2$

$\Rightarrow OP^2 + 576 = 625$

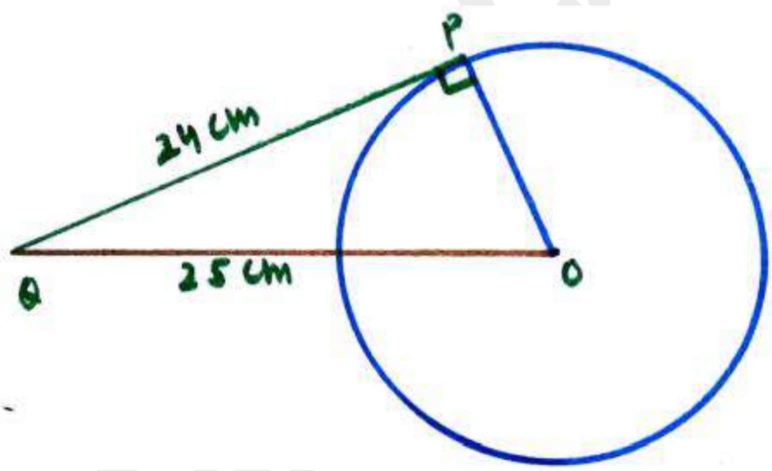
$\Rightarrow OP^2 = 625 - 576$

$\Rightarrow OP^2 = 49$

$\Rightarrow OP = 7$ cm

$\Rightarrow$ The radius of the circle is 7 cm.

$\therefore$ option (A) is correct.



2. In Fig. if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to

(A) 60° (B) 70°
(C) 80° (D) 90°

Sol. OP and OQ are radii of the circle to the tangents TP and TQ respectively.

$\therefore \angle OPT = \angle OQT = 90°$ each

In quadrilateral POQT,

$\angle PTQ + \angle OPT + \angle POQ + \angle OQT = 360°$

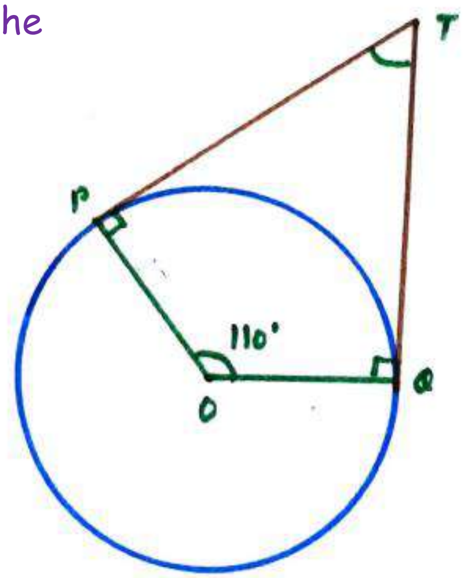
[Sum of angles of quadrilateral POQT]

$\Rightarrow \angle PTQ + 90° + 110° + 90° = 360°$

$\Rightarrow \angle PTQ = 360° - 290°$

$\Rightarrow \angle PTQ = 70°$

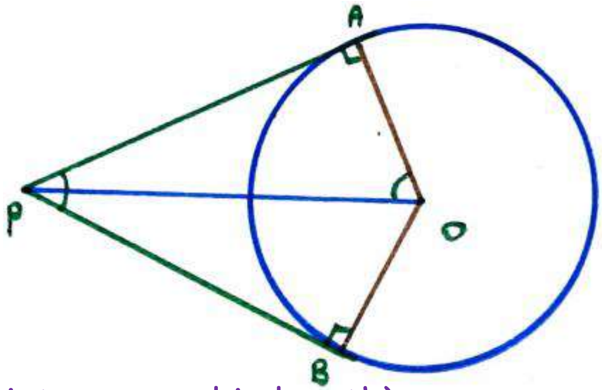
$\therefore$ option (B) is correct.



3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠ POA is equal to

(A) 50° (B) 60°
(C) 70° (D) 80°

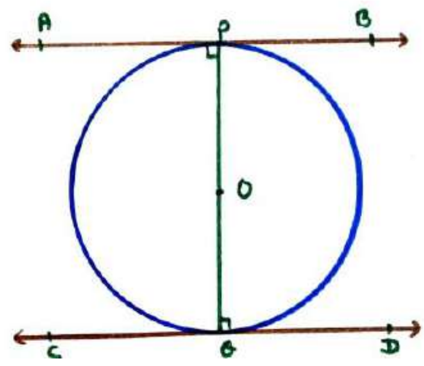
Sol. Since OA and OB are radii of the circle to the tangents PA and PB respectively.



∴ ∠PAO = ∠PBO = 90° each
In quadrilateral AOBP,
∠AOB + ∠PAO + ∠PBO + ∠APB = 360°
⇒ ∠AOB + 90° + 90° + 80° = 360°
⇒ ∠AOB = 360° - 260°
⇒ ∠AOB = 100°
Now, in Δ POA and Δ POB,
PA = PB (Tangents from an external point are equal in length)
OA = OB (Radii)
OP = OP (Common)
∴ Δ POA ≅ Δ POB (by SSS congruence condition)
∴ ∠POA = ∠POB (c.p.c.t.)
∠AOB = ∠POA + ∠POB
∠AOB = ∠POA + ∠POA (∵ ∠POA = ∠POB)
⇒ ∠AOB = 2 ∠POA
⇒ ∠POA = $\frac{1}{2}$ ∠AOB
⇒ ∠POA = $\frac{1}{2}$ 100°
⇒ ∠POA = 50°
∴ option (A) is correct.

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Sol. Let AB and CD are tangents at the ends of the diameter PQ respectively.



∴ OP ⊥ AB and OQ ⊥ CD
{ ∵ A and B are points of contact}
∴ ∠APQ = ∠DQO = 90°
But these are alternate angles.
So, AB and CD are parallel.
Hence the tangents drawn at the ends of a diameter of a circle are parallel.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Sol. Let AB be the tangent to the circle at point P with centre O.

Suppose the perpendicular at the point of contact to the tangent to a circle doesn't pass through the centre.

Let it passes through O'.

$\Rightarrow \angle O'PA = 90^\circ$

and $\angle OPA = 90^\circ$ (Radius is perpendicular at the point of contact with tangent)

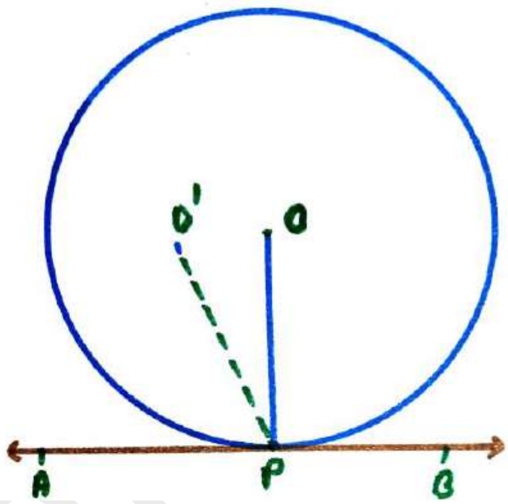
$\Rightarrow \angle O'PA = \angle OPA = 90^\circ$

It is possible only when O and O' are coincide

Which is a contradiction.

$\therefore$ Our supposition is wrong.

Hence, the perpendicular at the point of contact to the tangent to a circle passes through the centre. Proved.



6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Sol. AB is the tangent drawn on this circle from point A.

$\therefore OB \perp AB$

OA = 5cm and AB = 4 cm (Given)

In $\perp \Delta ABO$,

$AB^2 + BO^2 = OA^2$ [By Pythagoras theorem]

$\Rightarrow 4^2 + BO^2 = 5^2$

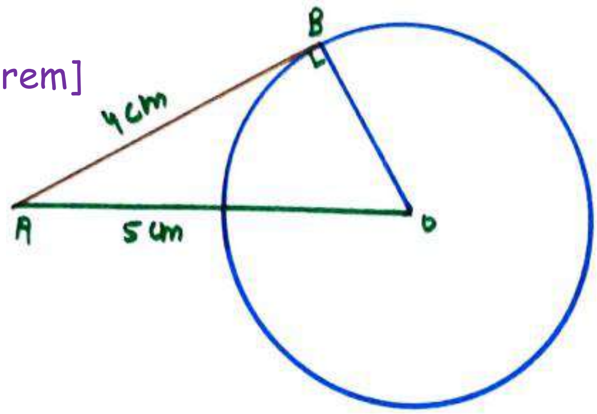
$\Rightarrow 16 + BO^2 = 25$

$\Rightarrow BO^2 = 25 - 16$

$\Rightarrow BO^2 = 9$

$\Rightarrow BO = 3$

$\therefore$ The radius of the circle is 3 cm. Ans.



7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Sol. Let AB is the chord of the larger circle which touches the smaller circle at point P.

∴ AB is tangent to the smaller circle.

⇒ OP ⊥ AB

∴ In ⊥ Δ AOP,

$AP^2 + OP^2 = OA^2$

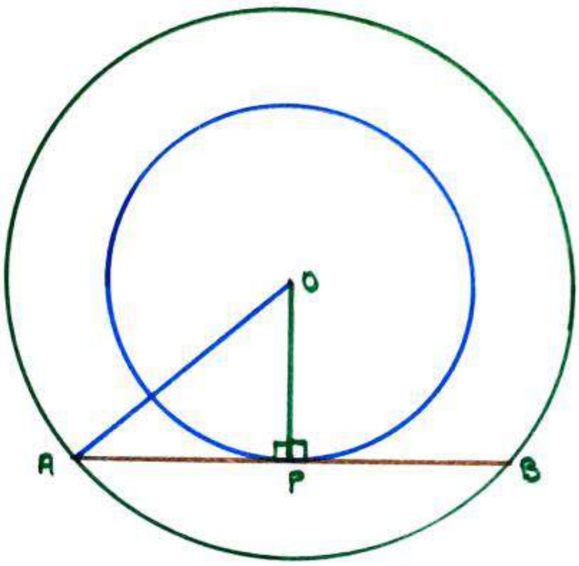
⇒ $AP^2 + 3^2 = 5^2$

⇒ $AP^2 + 9 = 25$

⇒ $AP^2 = 25 - 9$

⇒ $AP^2 = 16$

⇒ $AP = 4$



Since OP ⊥ AB,

⇒ AP = PB (∵ Perpendicular from the center of the circle bisects the chord)

⇒ AB = 2AP

= 2 × 4 = 8 cm

∴ The length of the chord of the larger circle is 8 cm. Ans.

8. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC

Sol. Since AP and AS are tangents from A

⇒ AP = AS (tangents from external point are equal in length)

Similarly, BP = BQ (…do….)

CQ = CR (…do….)

and DR = DS (…do….)

L. H. S. = AB + CD

= AP + BP + CR + DR

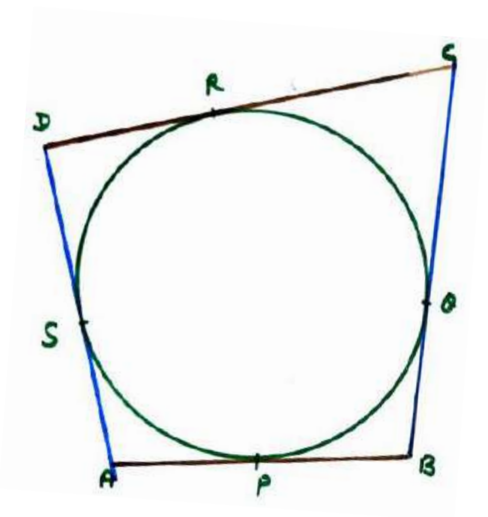
= AS + BQ + CQ + DS

= AS + DS + BQ + CQ

= AD + BC

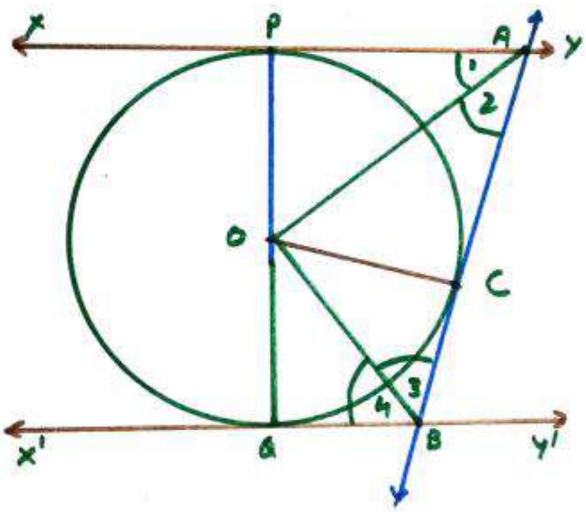
= R. H. S.

∴ AB + CD = AD + BC Proved.



9. In Fig. 10.13, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that ∠ AOB = 90°.

Sol. Joined O and C



In Δ AOP and Δ AOC,

OP = OC (Radii)

AP = AC (Tangents from the point A)

AO = AO (Common)

∴ ΔAOP ≅ ΔAOC (SSS congruency)

⇒ ∠1 = ∠2 (c.p.c.t.) … (i)

Similarly, in Δ BOQ and Δ BOC

OQ = OC (Radii)

BQ = BC (Tangents from the point A)

BO = BO (Common)

∴ Δ BOQ ≅ Δ BOC (SSS congruency)

⇒ ∠4 = ∠3 (c.p.c.t.) … (ii)

Now, ∠PAB + ∠QBA = 180 ° (Same side angles of transversal AB)

⇒ ∠1 + ∠2 + ∠3 + ∠4 = 180 °

⇒ ∠2 + ∠2 + ∠3 + ∠3 = 180 ° (∵ ∠1 = ∠2 and ∠3 = ∠4)

⇒ 2∠2 + 2∠3 = 180°

⇒ 2(∠2 + ∠3) = 180°

⇒ ∠2 + ∠3 = 90°

In Δ AOB, ∠2 + ∠3 + ∠AOB = 180°

⇒ 90° + ∠AOB = 180°

⇒ ∠AOB = 180° - 90°

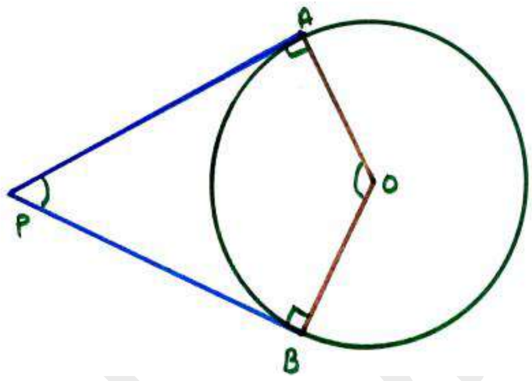
∴ ∠AOB = 90° Proved. Ans.

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Sol. Since PA and PB are tangents drawn from the external point P.

$\therefore \angle OAP = 90°$ and $\angle OBP = 90°$

(Angles at point of contact)



In quadrilateral OAPB,

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$

$\Rightarrow 90° + \angle APB + 90° + \angle BOA = 360°$

$\Rightarrow 180° + \angle APB + \angle BOA = 360°$

$\Rightarrow \angle APB + \angle BOA = 360° - 180°$

$\Rightarrow \angle APB + \angle BOA = 180°$

$\therefore$ The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre. Proved.

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. In parallelogram ABCD

AB = CD and BC = AD (opposite sides of parallelogram) ………………(i)

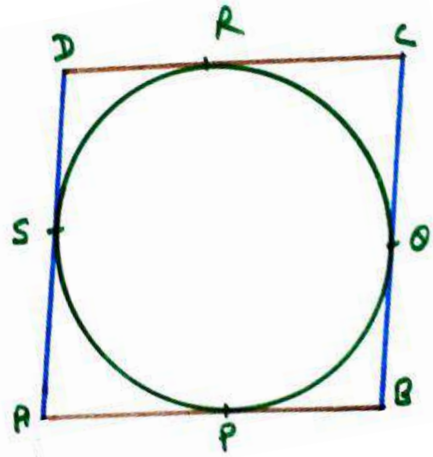
Since AP and AS are tangents from A

$\Rightarrow$ AP = AS (tangents from external point are equal in length)

Similarly, BP = BQ (…do….)

CQ = CR (…do….)

and DR = DS (…do….)



Now, AB + CD

= AP + BP + CR + DR

= AS + BQ + CQ + DS

= AS + DS + BQ + CQ

= AD + BC

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + AB = BC + BC ($\because$ AB = CD and BC = AD)

$\Rightarrow$ 2AB = 2BC

$\Rightarrow$ AB = BC ……………………… (ii)

From (i) and (ii)

$\Rightarrow$ AB = BC = CD = DA

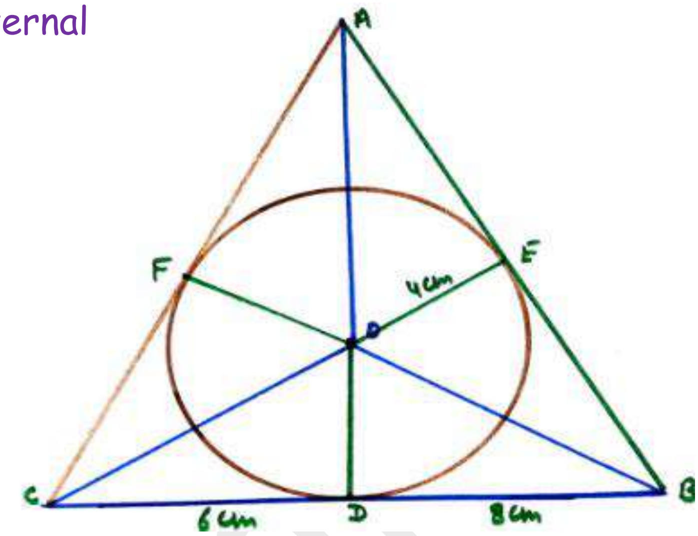
ABCD is a rhombus.

$\therefore$ The parallelogram circumscribing a circle is a rhombus. Proved.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.

Sol. In ABC, CF = CD = 6cm, BE = BD = 8cm and AE = AF = x (say)

(Tangents drawn from the external point are equal in length)



$\therefore$ AB = AE + EB = x + 8

BC = BD + DC = 8 + 6 = 14

And CA = CF + FA = 6 + x

Semi perimeter of triangle

$= \frac{AB + BC + CA}{2}$

$= \frac{x + 8 + 14 + 6 + x}{2}$

$= \frac{2x + 28}{2}$

= x + 14

$\therefore$Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)(x+14-x-8)(x+14-14)(x+14-x-6)}$

$= \sqrt{(x+14)(6)(x)(8)}$

$= \sqrt{48x(x+14)}$................................. ... (i)

also, Area of ΔABC =area of Δ AOB + area of Δ BOC + area of Δ COA

$= \frac{1}{2} \times AB \times OE + \frac{1}{2} \times BC \times OD + \frac{1}{2} \times CA \times OF$

$= \frac{1}{2} \times (x + 8) \times 4 + \frac{1}{2} \times 14 \times 4 + \frac{1}{2} \times (x + 6) \times 4$

= 2x + 16 + 28 + 2x + 12

= 4x + 56

= 4(x + 14) .... (ii)

From (i) and (ii) we get,

$\sqrt{48x(x+14)}$ = 4(x + 14)

Squaring both sides,

48x (x + 14 = 16 (x + 14)^2

$\Rightarrow$ 48x = 16 (14 + x)

$\Rightarrow$ 3 x = 14 + x

$\Rightarrow$ 2x = 14

$\therefore$ x = 7 cm

Hence, AB = x + 8 = 7 + 8 = 15 cm

CA = 6 + x = 6 + 7 = 13 cm Ans.

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol. Let ABCD be a quadrilateral circumscribing a circle with centre O such that it touches the circle at point P, Q, R, S.

Join the vertices of the quadrilateral ABCD to the center of the circle.

In ΔAOP and ΔAOS,

$AP = AS$ (Tangents from external point are equal in length)

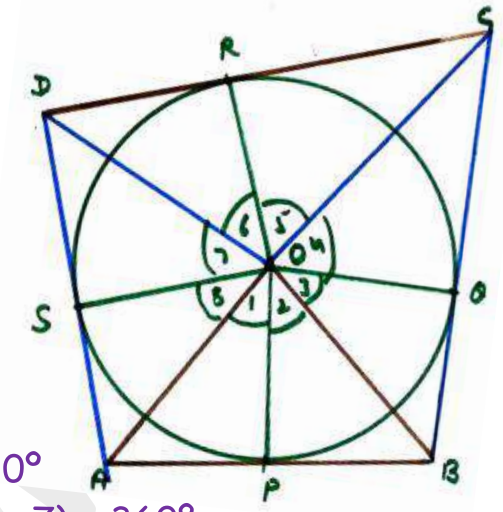
$OP = OS$ (Radii)

$OA = OA$ (Common)

$\Delta AOP \cong \Delta AOS$ (SSS congruency)

$\therefore \angle POA = \angle AOS$ (c.p.c.t.)

$\Rightarrow \angle 1 = \angle 8$



Similarly, we have

$\Delta BOP \cong \Delta BOQ$ and $\angle 2 = \angle 3$ (c.p.c.t.)

$\Delta COQ \cong \Delta COR$ and $\angle 4 = \angle 5$ (c.p.c.t.)

$\Delta DOR \cong \Delta DOS$ and $\angle 6 = \angle 7$ (c.p.c.t.)

Adding all these angles,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow (\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$

$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$

$\Rightarrow 2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$

Now, $\angle AOB + \angle BOC + \angle COD + \angle DOA = 360°$

$\angle AOB + \angle COD + \angle BOC + \angle DOA = 360°$

$180° + \angle BOC + \angle DOA = 360°$

$\angle BOC + \angle DOA = 360° - 180°$

$\angle BOC + \angle DOA = 180°$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. Proved.